BASIC ELECTRICAL ENGINEERING

UNIT-3

BALANCED THREE PHASE AC

Vibha Masti

Feedback/corrections: vibha@pesu.pes.edu

Three Phase AC

- ~400 to 440 V rms (line V)
- more power than single phase
- commercial loads: 3 phase
- agriculture, water pump, industry

Single Phase

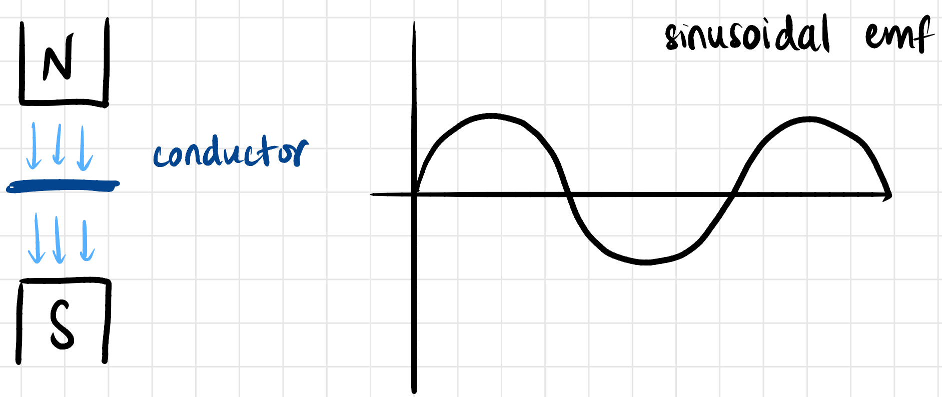
- line and neutral
- needs starting coil

Three Phase

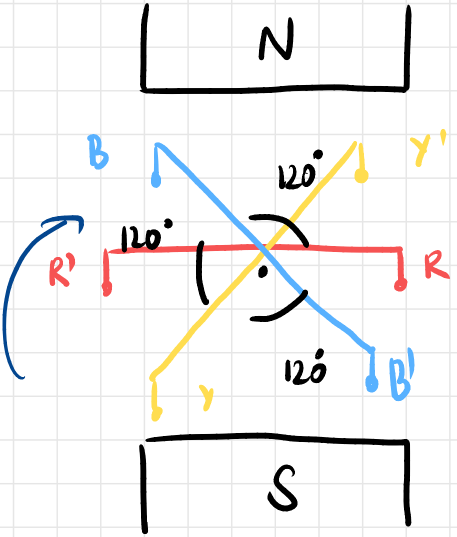
- 3 lines
- 3 voltages same magnitude, diff voltage
- polyphase system
- balanced system —— Z, V, I same, only ϕ diff
- can build 6, 12 phase systems
- rectifier output smooth for polyphase
- two-phase: 90° not 180° (fans)

Generate Single Phase Voltage

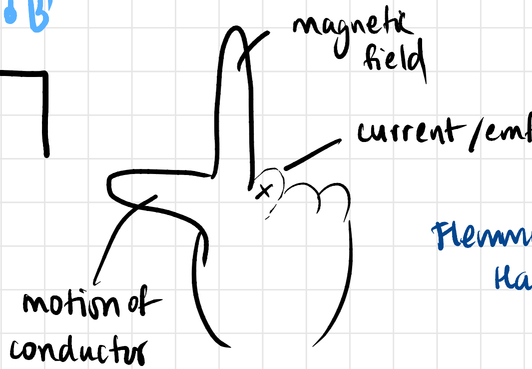
- Electromagnetic induction
- Rotate conductor in EM field



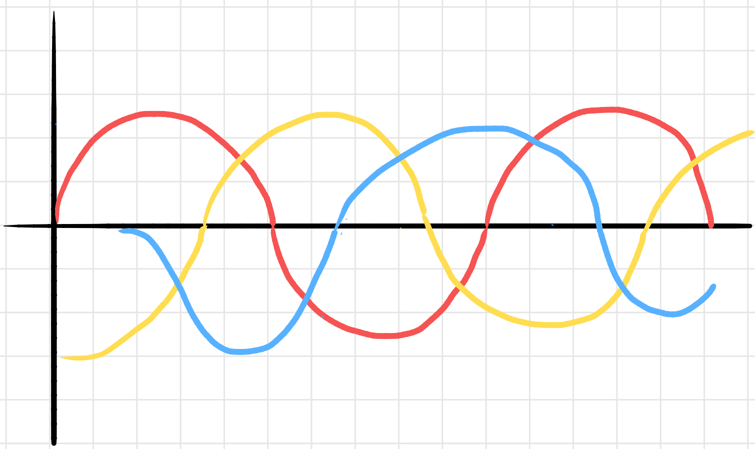
Generation of 3 Phase AC



rotate full system



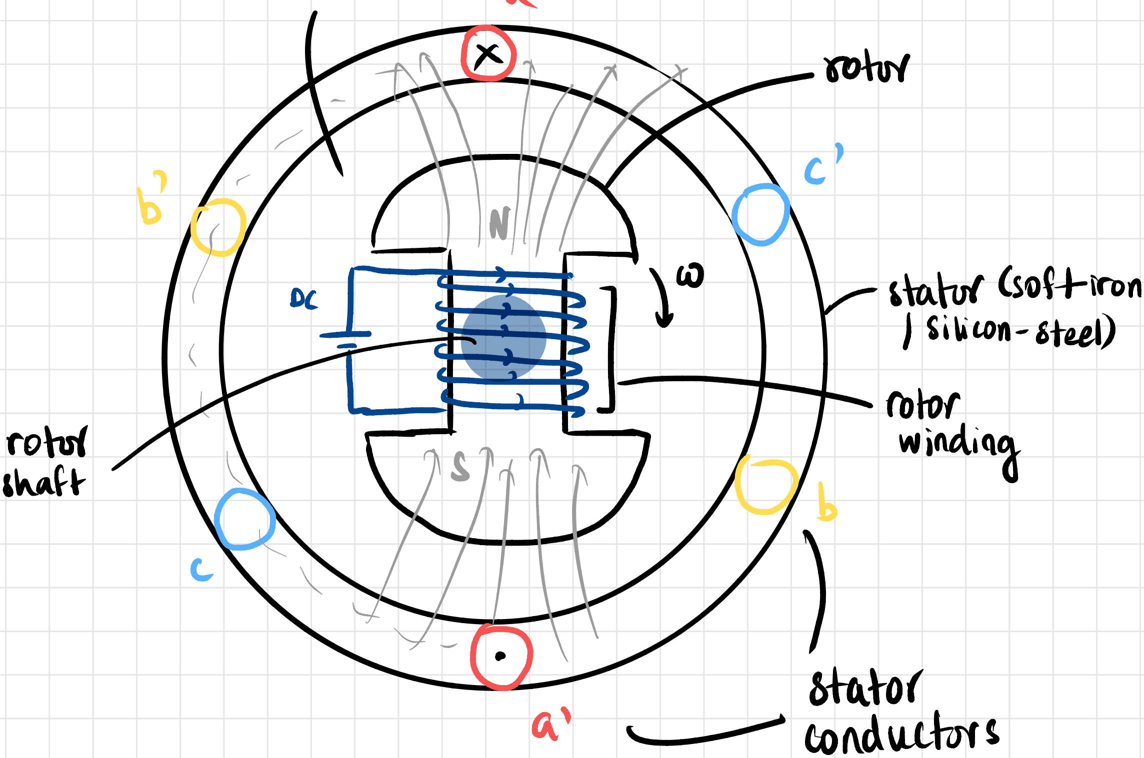
Flemming's Right Hand Rule



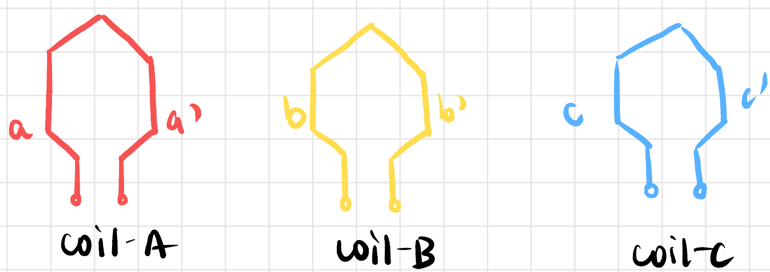
rough

ALTERNATOR | 3 PHASE AC GENERATOR

$$e \propto -\frac{d\phi}{dt}$$



- each coil placed 120° apart from each other



- Reluctance: opposition to flow of magnetic field lines (similar to resistance)

- Air gives more reluctance than soft iron; magnetic field flows through stator

- The generation and transmission of AC power is done as 3-phase AC

- Distribution of power to the industry is done as 3-phase AC.

- To the residential consumers, it is done as single phase AC.

- Generation of 3-phase emf occurs in a machine called 3 phase generator or alternator in a power generating station.

Constructional Details of Alternator

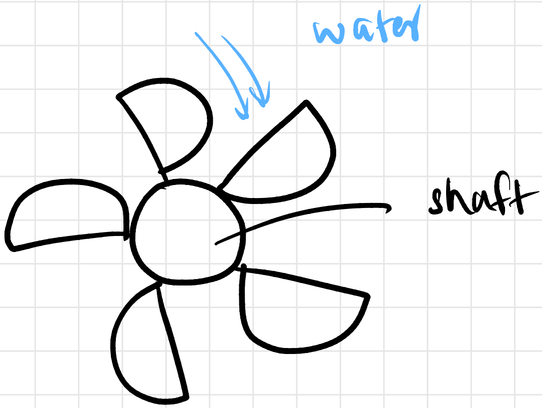
- The stator is a stationary member of this machine and it carries 3 coils which are physically displaced from one-another by 120°.

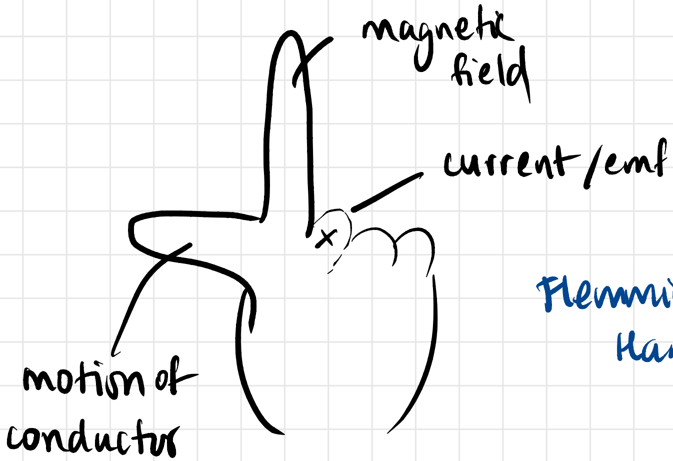
- Conductors a and a' make coil-A, which is also called as phase A; b and b' make coil-B and phase B, c and c' make coil-C and phase C

- The rotor of this machine consists of a set of electromagnets excited by a DC supply.

- The stator and rotor of this machine are usually made of silicon-steel.

Prime mover: turbine. shaft of turbine connected to shaft of generator.



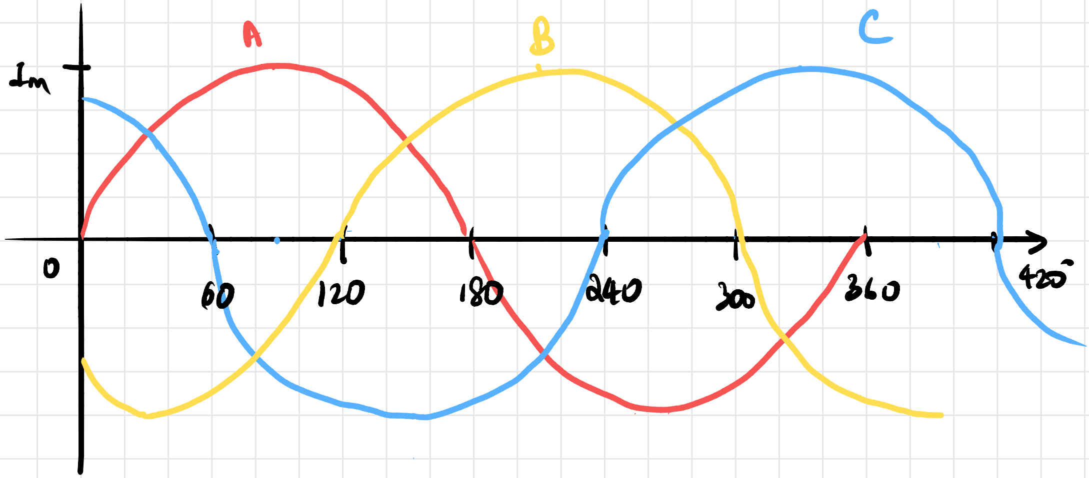


Flemming's Right Hand Rule

current generated is sinusoidal in nature.

Waveform

coil A as reference (starts at 0)



Also called R-Y-B (red-yellow-blue)

$e_{aa'} = V_m \sin \omega t$

$e_{bb'} = V_m \sin(\omega t - 120°)$

$e_{cc'} = V_m \sin(\omega t + 120°)$ or $V_{cc'} = E_m \sin(\omega t - 240°)$

Balanced 3-Phase

$e_{aa'} = 100 \sin \omega t$
$e_{bb'} = 100 \sin(\omega t - 120°)$
$e_{cc'} = 100 \sin(\omega t - 240°)$

] same magnitude, phase difference is 120°

Unbalanced 3 Phase System

$e_{aa'} = 100 \sin \omega t$
$e_{bb'} = 90 \sin(\omega t - 120°)$
$e_{cc'} = 100 \sin(\omega t - 240°)$
] different magnitude

$e_{aa'} = 100 \sin \omega t$
$e_{bb'} = 100 \sin(\omega t - 90°)$
$e_{cc'} = 100 \sin(\omega t - 240°)$
] different phase

Power Generated

- In power stations, 3 phase balanced power is generated.
- To supply to industry, 3-phase distributed
- For household power, single phase power is distributed.

RYB Representation

$e_{RR'} = V_m \sin \omega t$
$e_{YY'} = V_m \sin(\omega t - 120°)$
$e_{BB'} = V_m \sin(\omega t - 240°)$

Phase Sequence

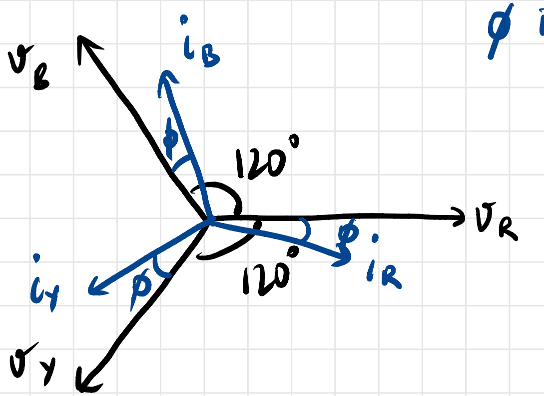
Phase sequence is always referred to in a 3-phase supply.

It is the order in which the three phases reach their maximum values

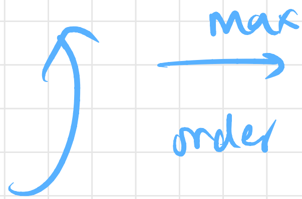
If the maximum values of three phase voltages occur in the order RYB, the phase sequence is RYB.

RYB

$v_R \angle 0$
$v_Y \angle -120$
$v_B \angle -240$ or $v_B \angle 120$



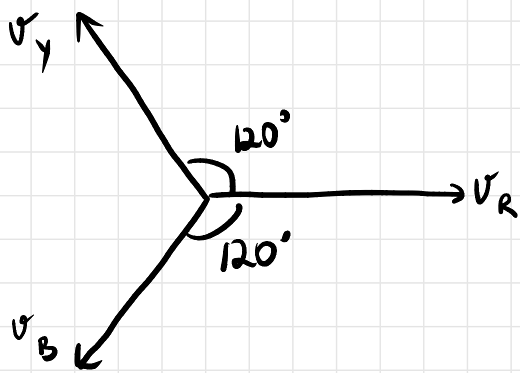
ϕ is same



Otherwise, if the maximum value of three phase voltages occur in the order RBY, the phase sequence is RBY.

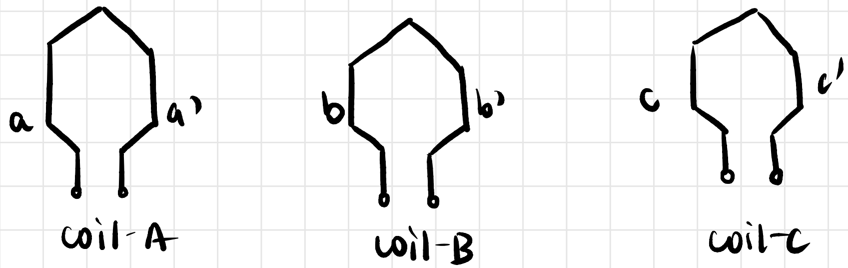
RBY

$v_R \angle 0$
$v_B \angle -120$
$v_Y \angle -240$

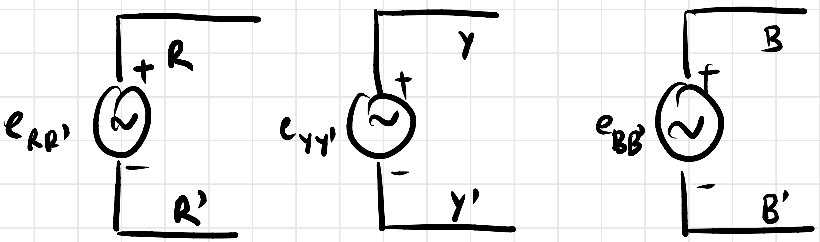


Connections of 3-Phase System

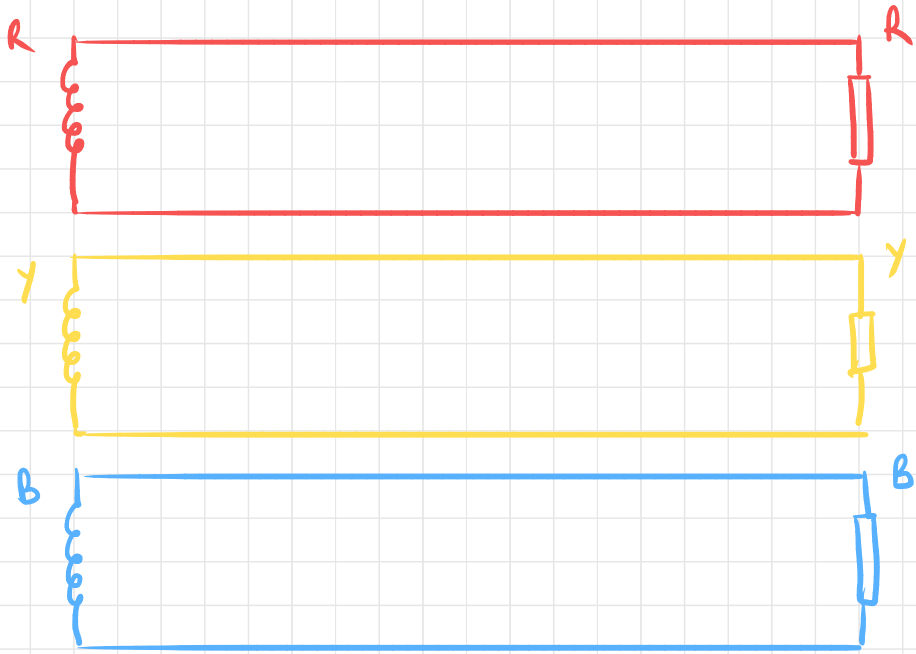
1) Star
2) Delta



Three single-phase systems in the three coils



Possible Connection



To avoid transmission infrastructure and costs, either star or delta connection is used (6 lines)

Delta allowed : KVL
Star allowed : KCL

STAR CONNECTED SYSTEM

We connect the same terminals of all the three phases together to a common point which is called the neutral point.

Phase Voltage

Voltage across the terminals of a phase in a 3-phase system

e_{ph} or V_{ph}

(line to ground)

Line Voltage

Voltage between any two lines in a 3-phase system is called the line voltage.

E_L or V_L

(line to line)

Phase current

current flowing through a phase in a three-phase system

i_{ph}

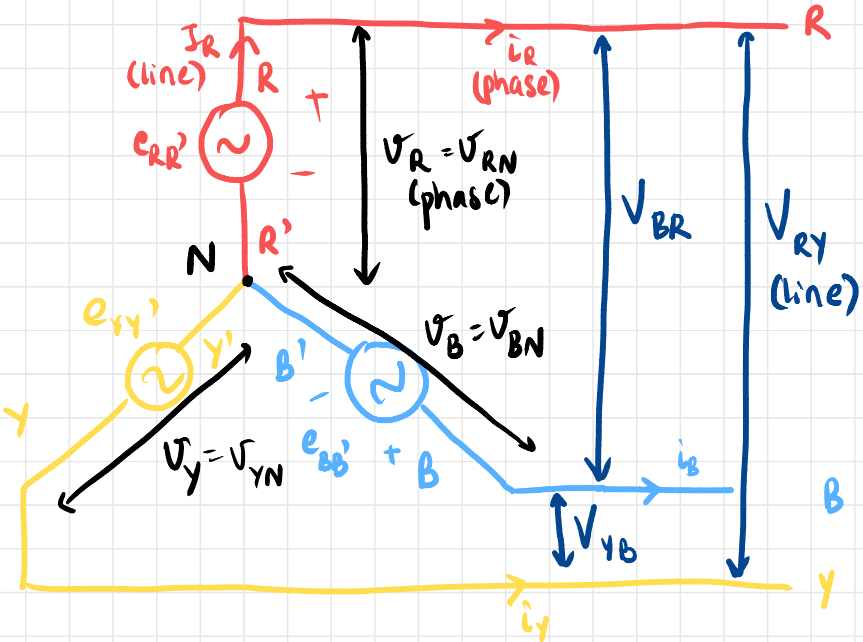
Line current

current flowing in a line is called a line current

I_L

Relationship Between Line & Phase Quantities

$e_{RR'} = V_m \sin \omega t$
$e_{YY'} = V_m \sin(\omega t - 120°)$
$e_{BB'} = V_m \sin(\omega t + 120°)$



(centre can also be RYB)

Phase & Line Quantities (balanced 3ϕ)

$I_R = I_Y = I_B = I_L$ (line current)

$V_{RY} = V_{YB} = V_{BR} = V_L$ (line voltage)

$i_R = i_Y = i_B = i_{ph}$ (phase current)

$v_R = v_Y = v_B = v_{ph}$ (phase voltage)

Phase and Line Currents

$I_R = i_R$

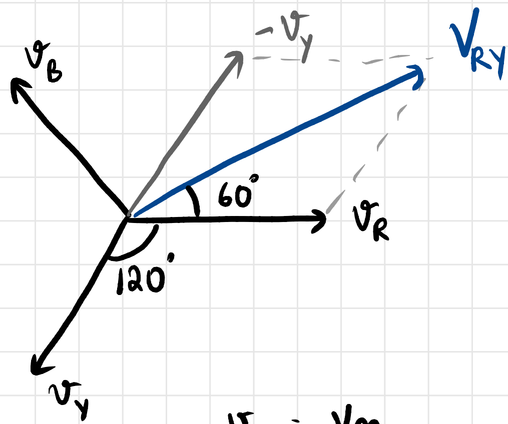
$I_L = i_{ph}$ → line current = phase current

Phase and Line Voltages

Apply KVL on RYNR

$-\bar{V}_{RY} - \bar{v}_Y + \bar{v}_R = 0$

$\bar{V}_{RY} = \bar{v}_R - \bar{v}_Y$



$\bar{V}_L = \bar{v}_R - \bar{v}_Y$

$\bar{V}_L = \frac{V_m}{\sqrt{2}} \angle 0 - \frac{V_m}{\sqrt{2}} \angle -120$

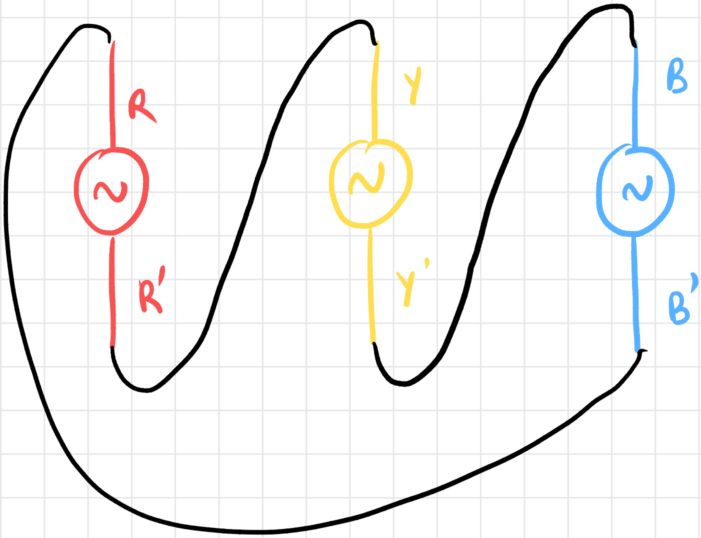
$v_{ph} = \frac{V_m}{\sqrt{2}}$

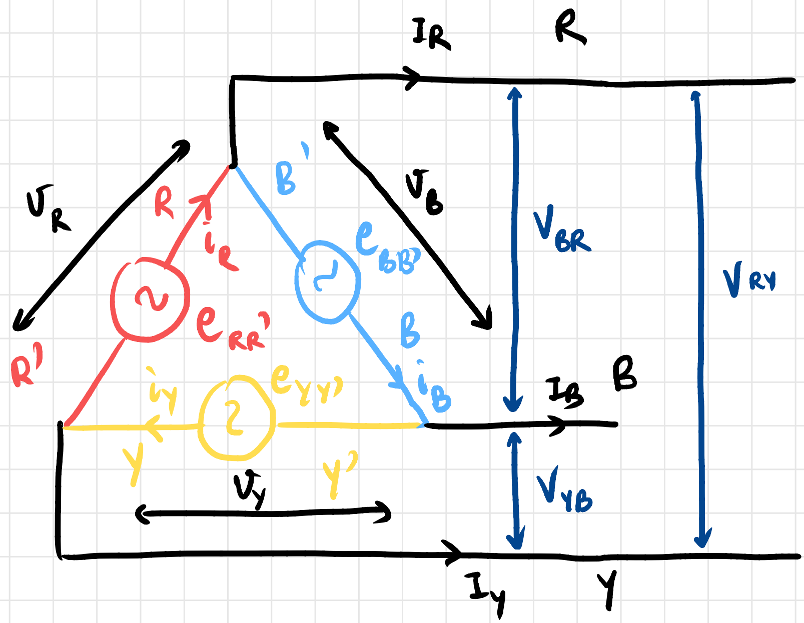
$\bar{V}_L = \frac{V_m}{\sqrt{2}} \sqrt{3} \angle 30$

$$V_L = \sqrt{3}\, v_{ph} \angle 30$$

Line voltage leads phase voltage by 30°

DELTA CONNECTED SYSTEM





- In Δ connection, second terminal of first coil is connected to the first terminal of the second coil, and so on.
- There is no neutral point

Phase & Line Quantities (balanced 3ϕ)

$I_R = I_Y = I_B = I_L$ (line current)
$V_{RY} = V_{YB} = V_{BR} = V_L$ (phase current)

$i_R = i_Y = i_B = i_{ph}$ (phase current)
$v_R = v_Y = v_B = v_{ph}$ (phase voltage)

Phase and Line Voltages

$$V_L = v_{ph}$$

Phase and Line Currents

Applying KCL at juction R

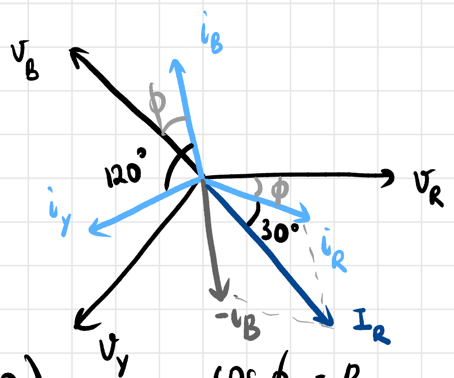
$$\bar{i}_R = \bar{i}_B + \bar{I}_R$$

$$\bar{I}_R = \bar{i}_R - \bar{i}_B$$

$$\bar{I}_L = \frac{\bar{v}_R}{Z} - \frac{\bar{v}_B}{Z}$$

$$= \frac{1}{Z} \frac{V_m}{\sqrt{2}} \left(1\angle 0 - 1\angle 120 \right)$$

$$= \frac{I_m}{\sqrt{2}} \sqrt{3} \angle -30$$



$\cos\phi = \frac{R}{Z}$

$$\bar{I}_L = \sqrt{3}\, i_{ph} \angle -30$$

Line current lags phase current by 30°

- default: assume line quantity given

POWER CONSUMED (3-ϕ)

Active Power (P)

$$P = 3 \times \text{1-phase power}$$
$$= 3 \times \text{power in each phase}$$
$$= 3 \, v_{ph} \, i_{ph} \cos\phi$$

$$P_{3p} = 3 \times \frac{V_L \, I_L}{\sqrt{3}} \cos\phi$$

$$P_{3p} = \sqrt{3} \, V_L \, I_L \cos\phi$$

Reactive power (Q)

$$Q_{3p} = 3 \, v_{ph} \, i_{ph} \sin\phi = \sqrt{3} \, V_L \, I_L \sin\phi$$

$$\cos\phi = \frac{R}{Z}$$

Apparent Power (S)

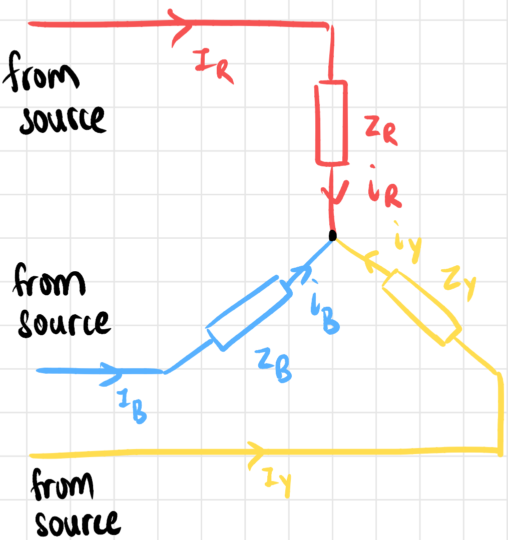
$$S_{3p} = \sqrt{3} \, V_L \, I_L$$

Load connection

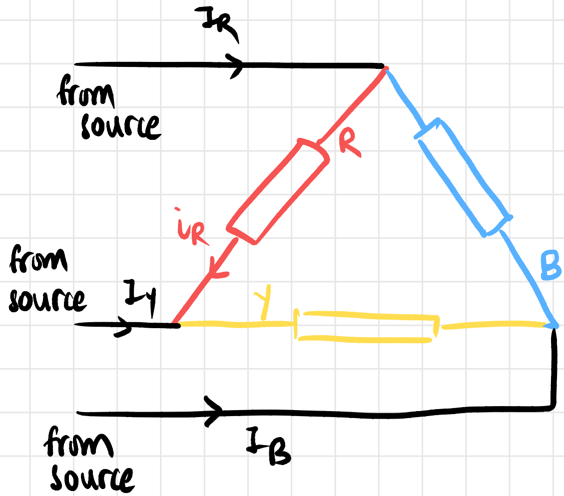
if $z_1 = z_2 = z_3 = z$, balanced load system

else, unbalanced

star (Y)



delta (Δ)



Lines connect source to load.

Currents that enter: load current
Currents that exit: source currents

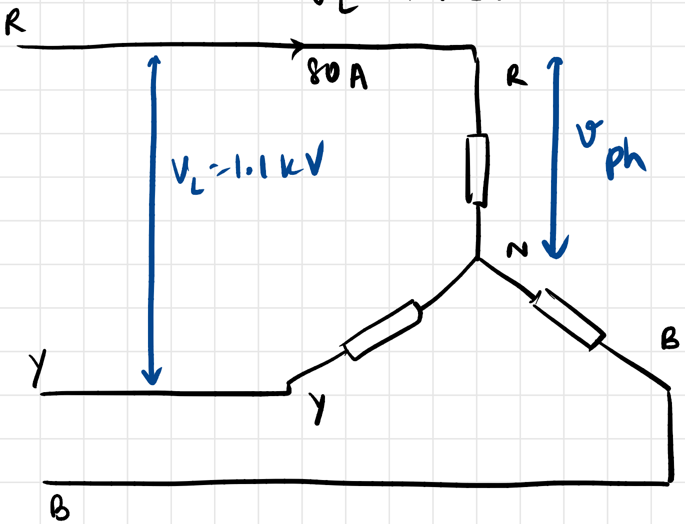
Line values: capital (I_L, V_L)] RMS
Phase values: lowercase (i_{ph}, v_{ph})

Q: A balanced 3ϕ Y load of 100 kW takes a leading current of 80 A when connected to 3ϕ 1.1 kV, 50 Hz supply. Find R, Z, C of load per phase. Also calculate PF of load.

$$P = 100\,kW = \sqrt{3} V_L I_L \cos\phi$$

$I_L = 80\,A$

$V_L = 1.1\,kV$



$P = 3 i_{ph}^2 R$

$100 = 3 I_L^2 R$

$R = 5.208\,\Omega$

$V_L = \sqrt{3} V_{ph}$

$V_{ph} = 635\,V$

$Z = \frac{V_{ph}}{i_{ph}} = 7.938\,\Omega$

$Z = 5.208 - jX_L \Rightarrow X_c = 5.99\,\Omega$

$X_c = \frac{1}{\omega C} \Rightarrow C = \frac{1}{2\pi f X_c}$

$C = 531.4\,\mu F$

power factor $= \cos\phi = \frac{R}{Z} = 0.66$ lead

Q: A balanced 3ϕ star-connected load is supplied with a symmetrical 3ϕ 400 V system. The current in each phase is 30 A and lags by 30° by voltage

(i) Find impedance in each phase

(ii) Total power drawn

(iii) Phasor diagram

(i) $E = 400V = E_{line}$

$$V_L = \sqrt{3}\, v_{ph}$$

$i_{phase} = 30A$

$\theta = 30°$ lag

$$v_{phase} = \frac{400}{\sqrt{3}} = 230.94\ V$$

$$i_{phase} = 30 \angle -30°$$

$$Z_{phase} = 7.698 \angle 30 = \frac{40\sqrt{3}}{9} \angle 30$$

(θ ← 30)

$i_R = 30 \angle -30°$

$i_Y = 30 \angle -120°$

$i_B = 30 \angle 90°$

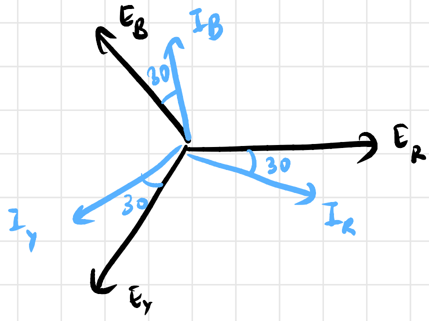
$Z_R = 7.698 \angle 30$

$Z_Y = 7.698 \angle 150$

$Z_B = 7.698 \angle -90$

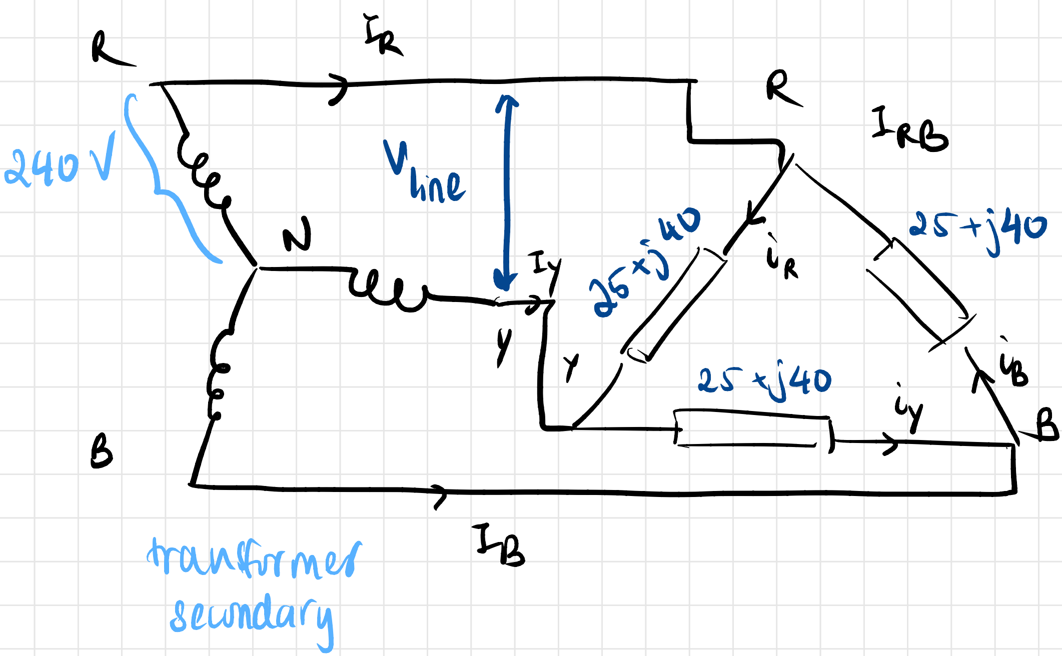
(ii) Power drawn = $3 \times V_{ph} I_{ph} \cos\theta$ = 18 kW

(iii) Phasor diagram



Q: A 3ϕ Δ connected load. Each phase Z = 25 + j40 Ω. The load is fed from the secondary of a 3ϕ Y-connected transformer with V_{phase} = 240V. Draw the circuit diagram and calculate

(i) Current in each phase of the load
(ii) Voltage across each phase of the load
(iii) Current in the transformer secondary winding
(iv) Power supplied to the load.



line voltage of Y transformer = phase voltage of Δ load

In Y system

$$V_L = v_{ph} \sqrt{3} \angle 30$$

$$\sqrt{3}\, v_{ph(Y)} = V_{L(Y)} = 240\sqrt{3}$$

$$V_L = 415.69 \angle 30^\circ$$

$$v_{ph(\Delta)} = 415.69 \angle 30$$

$$i_{ph(\Delta)} = \frac{v_{ph(\Delta)}}{Z} = 8.81 \angle -58 \text{ A}$$

Phase voltages

$v_R = 415.69 \angle 30$
$v_Y = 415.69 \angle -90$
$v_B = 415.69 \angle -210$

Phase current

$i_R = 8.81 \angle -28$
$i_Y = 8.81 \angle -148$
$i_B = 8.81 \angle -268$

Line current

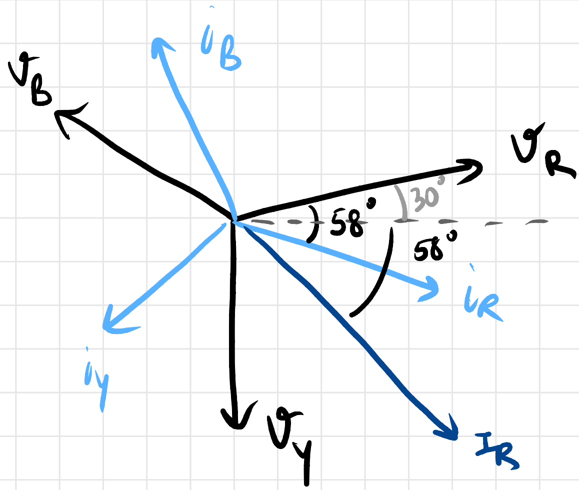
$$I_L = \sqrt{3}\, i_{ph} \angle -30$$

$15.26 \angle -58$
$15.26 \angle -178$
$15.26 \angle -298$

power

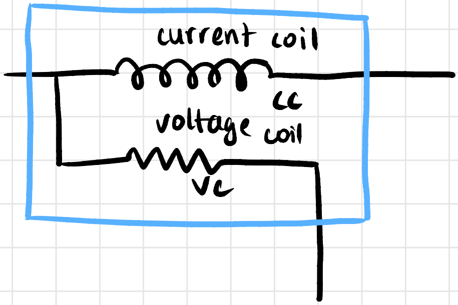
$$3 \times i_{ph}^2 R = 5.8 \text{ kW}$$

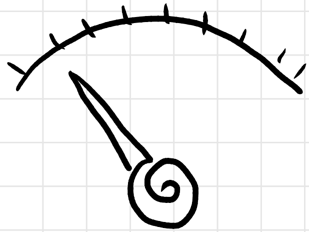
Phasor Diagram



TWO WATTMETER METHOD OF MEASURING 3-ϕ POWER

- measuring device that measures power





- two coils: fixed coil → gives magnetic flux
  voltage coil → indicates the power

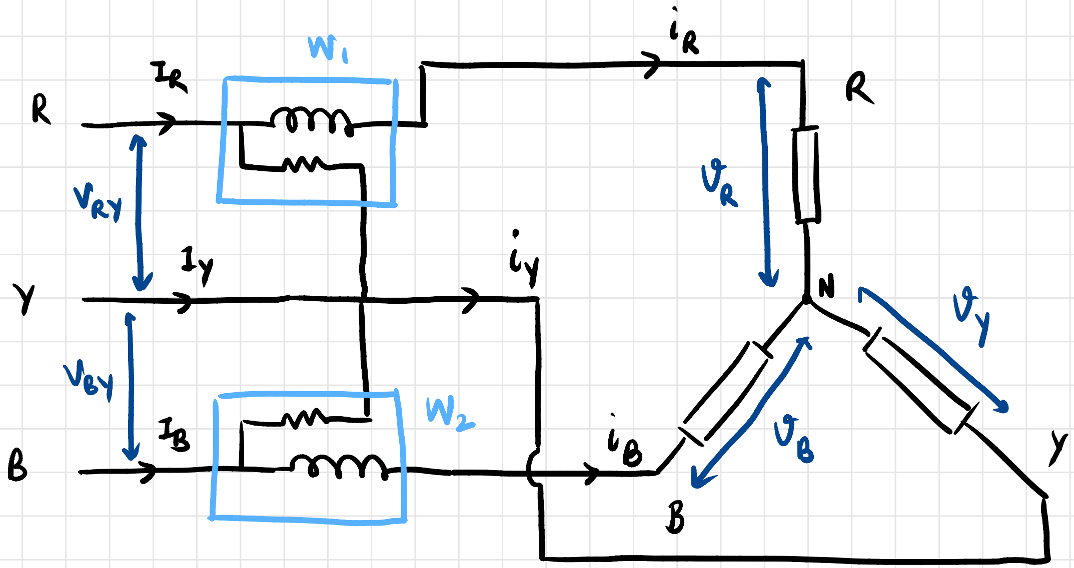
Construction

1. TWs are sufficient to find total active power in a 3-ϕ system
2. Irrespective of whether load in Y or Δ connected and whether load is balanced / unbalanced
3. A wattmeter is a power measuring device to measure 3-ϕ power. Consists of 2 coils (cc and vc).
4. The cc is a fixed coil which creates the magnetic field necessary for the operation of the instrument.
5. The vc or pressure coil is the movable coil which has an indicator attached to it.

Working

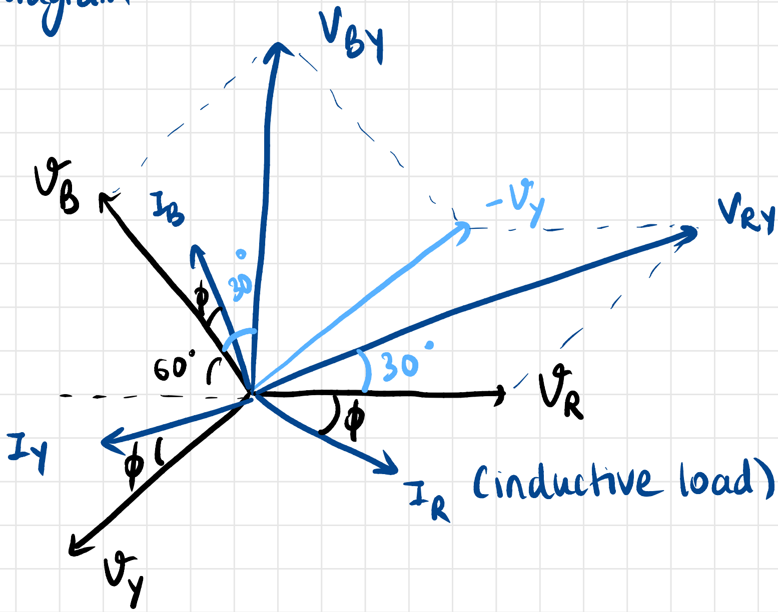
1. As the current flows through the instrument, the current in the current coil creates the magnetic field and current in the voltage coil interacts with this $\vec{B}$ and experiences a force.

2. This force moves the voltage coil and the indicator attached to it and points to the average power reading



- consider any one line as a common line
- Line currents: I_R, I_Y, I_B
- Phase currents: i_R, i_Y, i_B
- Power measured by $W_1 = V_{RY} I_R \cos$ (angle b/w V_{RY} & I_R)
- Power measured by $W_2 = V_{BY} I_B \cos$ (angle b/w V_{BY} & I_Y)

Phasor diagram



Power measured by $W_1 = V_{RY} I_R \cos$ (angle b/w V_{RY} & I_R)

$= (V_R - V_Y) I_R \cos(\text{angle})$

$$W_1 = V_{RY} I_R \cos(30+\phi)$$

$$W_1 = V_L I_L \cos(30+\phi)$$

Power measured by $W_2 = V_{BY} I_Y \cos$ (angle b/w V_{BY} & I_Y)

$= V_{BY} I_Y \cos(30-\phi)$

$$W_2 = V_L I_L \cos(30-\phi)$$

Total power: $W_1 + W_2 = V_L I_L \cdot 2\cos(30)\cos(\phi)$

$= \sqrt{3} V_L I_L \cos\phi$

Power factor

$$W_2 - W_1 = V_L I_L \cdot 2 \cdot \sin 30 \sin\phi$$
$$= V_L I_L \sin\phi$$

$$\frac{W_2 - W_1}{W_2 + W_1} = \frac{1}{\sqrt{3}} \tan\phi$$

$$\tan\phi = \sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{3}(W_2 - W_1)}{W_2 + W_1}\right)$$

$$W_1 < W_2$$

For capacitive load

$$W_1 = V_L I_L \cos(30 - \phi)$$
$$W_2 = V_L I_L \cos(30 + \phi)$$

$$W_2 < W_1$$

ϕ	P.F	$W_1 = V_L I_L \cos(30+\phi)$	$W_2 = V_L I_L \cos(30-\phi)$	$P_{3ph} = \sqrt{3} V_L I_L \cos\phi$	Remarks
0	1	$\frac{\sqrt{3}}{2} V_L I_L$	$\frac{\sqrt{3}}{2} V_L I_L$	$\sqrt{3} V_L I_L$	$W_1 = W_2$ load is resistive

ϕ	P.F	$W_1 = V_L I_L \cos(30+\phi)$	$W_2 = V_L I_L \cos(30-\phi)$	$P_{3ph} = \sqrt{3} V_L I_L \cos\phi$	Remarks
30	0.866 lag	$\frac{V_L I_L}{2}$	$V_L I_L$	$\frac{3}{2} V_L I_L$	$W_2 = 2W_1$
60	0.5 lag	0	$\frac{\sqrt{3} V_L I_L}{2}$	$\frac{\sqrt{3} V_L I_L}{2}$	$W_1 = 0$ $W_2 = P_{3ph}$
>60	<0.5	-ve	+ve		$W_1 = -ve$ $W_2 = +ve$ interchange any one coil's terminals in W_1

Q: In TWM, the ratio of readings is 3:1. Find PF

$$\tan\phi = \frac{\sqrt{3}(W_2 - W_1)}{W_2 + W_1} = -\frac{\sqrt{3}(2)}{4} = -\frac{\sqrt{3}}{2}$$

$$\cos\phi = \cos\left(\tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$$

$$= 0.76 \text{ lead}$$

Q:- Total power and reading of 2 WM = ?   P = ? W_1 = ? W_2 = ?

Reactive power = 15 KVAR = Q
Load power factor = 0.8 lag

$Q = S \sin\phi = 15$ KVAR

$\cos\phi = 0.8$ (inductive)
$\phi = 36.87°$

$15 = S \sin 36.87$

$S = 25$ KVA

$P = 25 \cos\phi = 20$ kW

$\sqrt{3} V_L I_L = 25$
$V_L I_L = \frac{25}{\sqrt{3}}$

$W_1 = V_L I_L \cos(30 + 36.87) = 5.67$ kW

$W_2 = V_L I_L \cos(-6.87) = 14.33$ kW

Q. Two wattmeters are connected to measure power in a 3ϕ circuit. The reading of one of the WM is 5kW when load PF is unity. The PF of load is changed to 0.707 lag without changing total input power. Calculate the readings of two wattmeters

$$W_1 = 5\text{ kW} = W_2 \quad = \frac{\sqrt{3}}{2} V_L I_L \Rightarrow V_L I_L = \frac{10}{\sqrt{3}}$$
$$PF = 1 \Rightarrow \phi_1 = 0$$

$$P = \sqrt{3} V_L I_L$$

$$\text{total power} = \sqrt{3} V_L I_L = \frac{10}{\sqrt{3}} \times \sqrt{3} = 10\text{ kW}$$

$$\cos\phi_2 = 0.707 \text{ lag (inductive)}$$

$$\phi_2 = 45^\circ$$

$$W_1 + W_2 = 10\text{ kW}$$

$$P = V_L I_L(\cos(75) + \cos 15) = 10$$

$$V_L I_L \frac{\sqrt{6}}{2} = 10$$

$$V_L I_L = \frac{10\sqrt{6}}{3} = 8.16$$

$$W_1 = V_L I_L \cos 75 = 2.11\text{ kW}$$

$$W_2 = V_L I_L \cos 15 = 7.89\text{ kW}$$

$$P = S\cos\phi \Rightarrow S = 10\sqrt{2}\text{ KVA}$$
$$Q = S\sin\phi = 10\sqrt{2} \times \frac{1}{\sqrt{2}} = 10\text{ KVAR}$$

Q: A 3ϕ star-connected load draws a line current of 20 A. The load KVA and kW are 20 and 11 respectively. Find the readings on each of the two wattmeters used to measure the 3ϕ power.

$$S = 20 \text{ kVA}$$
$$P = 11 \text{ kW}$$
$$I_{line} = 20A$$

$$P = S \cos\phi$$
$$11 = 20 \cos\phi$$

$$\phi = 56.63^\circ$$

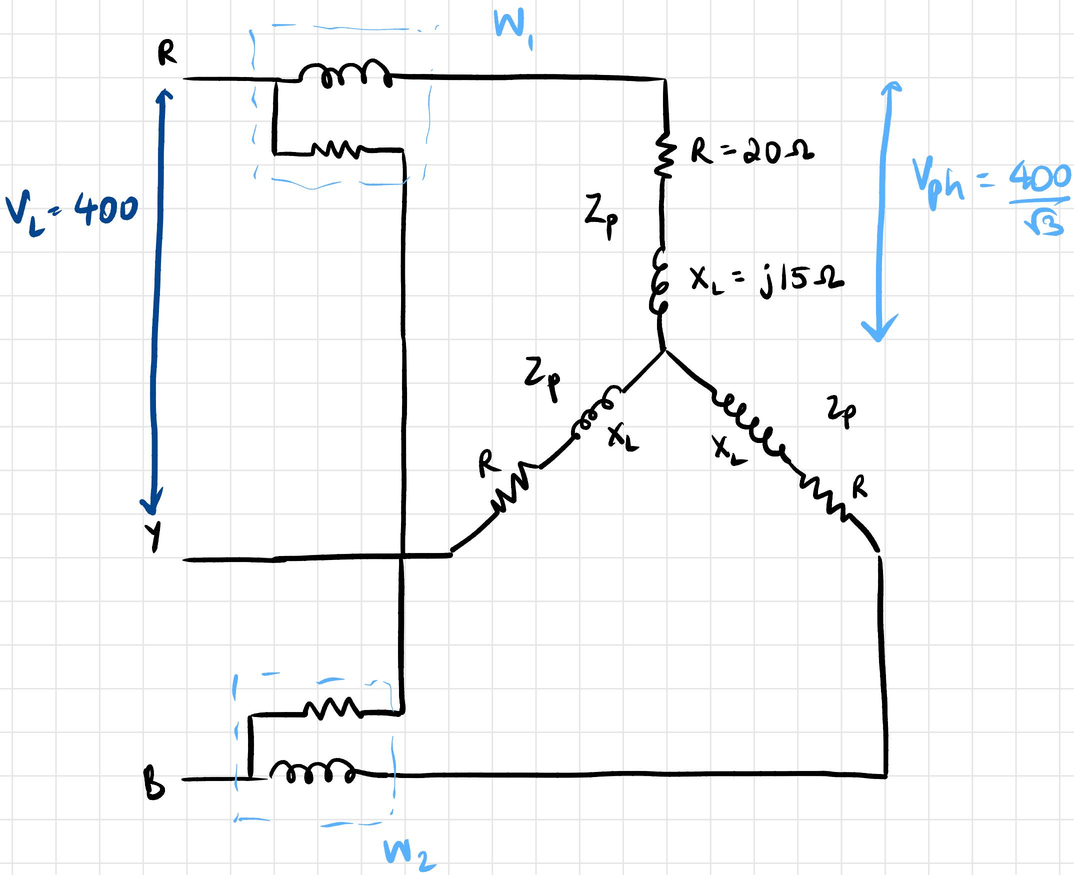
$$S = \sqrt{3} V_L I_L = 20 \Rightarrow V_L I_L = \frac{20}{\sqrt{3}}$$

$$W_1 = V_L I_L \cos(30 + 56.63)$$
$$= \frac{20}{\sqrt{3}} (0.0587) = 0.68 \text{ kW}$$

$$W_2 = V_L I_L \cos(-26.63)$$

$$= \frac{20}{\sqrt{3}} (0.8939) = 10.32 \text{ kW}$$

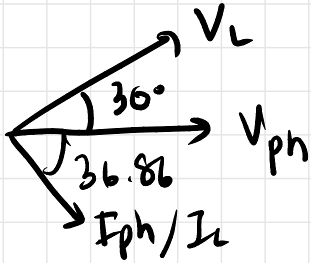
Q: Calculate the readings of two wattmeters connected to measure the total power for a balanced Y connected load as shown in the figure, fed from a 3ϕ 400 V balanced supply with phase sequence of RYB. Also find readings of the meters if they are connected in Δ.



$$Z_{ph} = 20 + j15 = 25\angle 36.87$$

$$I_{ph} = 9.24\angle -36.87 \text{ A} \Rightarrow I_L = 9.24$$

$$\phi = -36.87 \text{ (lag)}$$



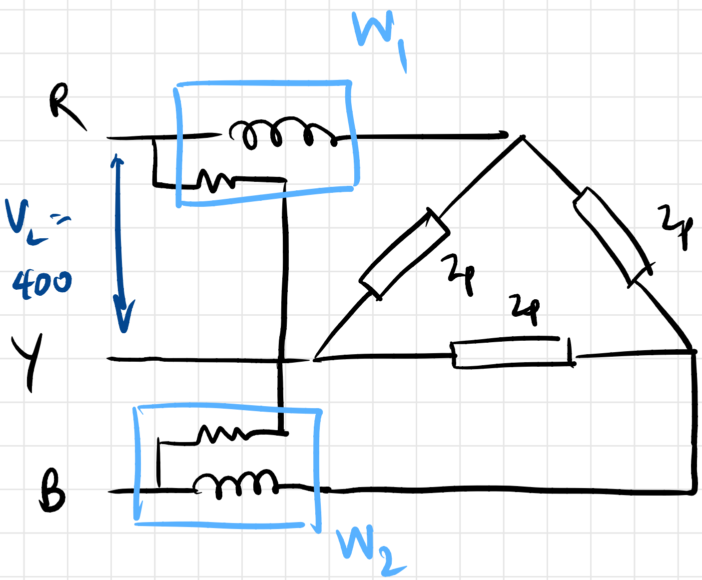
$\phi = 36.87$

1.59
4
MW ?

$W_1 = V_L I_L \cos(66.87) = 1.45$ kW

$W_2 = V_L I_L \cos(-6.87) = 3.67$ kW

In Δ



$I_{ph} = 16 \angle -36.87$
$V_{ph} = 400V$
$V_L = 400V$

$I_L = \sqrt{3} I_{ph}$

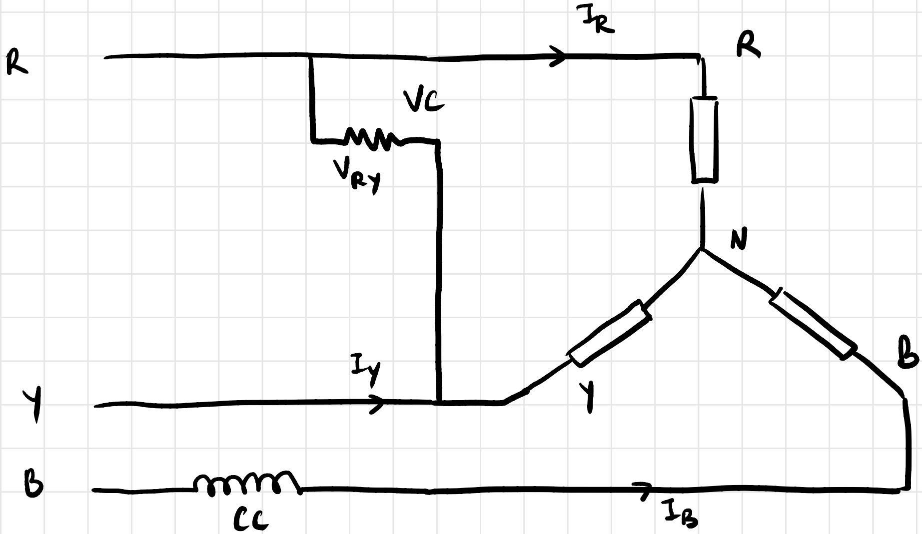
$I_L = 27.71 \angle -66.87$

$\phi = 36.87$

$W_1 = V_L I_L \cos(30 + 36.87) = 4.35$ kW

$W_2 = V_L I_L \cos(-6.87) = 11.00$ kW

Q: The potential coil of dynamo type wattmeter is connected from R to Y terminal of the load. The current coil of the meter is connected in series with phase B. By appropriate circuit diagram, show that the quantity indicated by this wattmeter is proportional to the reactive power drawn by the load. The phase sequence is RYB.



$Q = \sqrt{3}\, V_L I_L \sin\phi$

V_L = VC reading
I_L = CC reading

$W_1 = V_{RY} I_B \cos(90 - \phi)$

$W_1 = V_{RY} I_B \sin\phi$

$W_1 = V_L I_L \sin\phi$

$W_1 \propto Q$

Phasor Diagram